

## AN INTEGRATION APPROACH TO REDERIVE THE GREEN AND AMPT WATER INFILTRATION EQUATION AND ITS EXPERIMENTAL EVALUATION

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### ABSTRACT

The approach of the Green and Ampt (1911) for describing water entry into unsaturated porous media was critically appraised. Theoretical revision was made of a previous mathematical approach, to give it complete rigor in describing water flow for rectangular abrupt water content profile. Rigorous rederivation of this equation yielded a straightforward and complete interpretation of the suction head at the wet front  $\tau_w$ . The suction head  $\tau_w$  at the wet front is given as the integral of the suction-head-  $\tau$  dependent hydraulic conductivity function  $K(\tau)$  with the upper limit of the integral being the suction head  $\tau_0$  at the initial water content  $\theta_0$ . Furthermore,  $\tau_w$  is shown to be a constant of the flow process even when G-A rectangular abruptness does not hold. The equation is shown to be the hydraulic conductivity  $K(\theta_1)$  of the wet region behind the wet front which is a function of the constant averaged water content of the wet region. For experimental data of cumulative infiltration with time, both the G-A and two-term Philip equation were fitted by nonlinear regression. The G-A equation gave better fitting than Philip equation for certain soil texture, but both equations gave a very well fitting (%1error). G-A parameter  $K_1$  gave a very good agreement with the measured hydraulic conductivity for all soil studied. The other parameter  $\tau_w$  was calculated from the fitted parameter and showed values correlated to soil texture in expected. Abruptness of soil water transient profile was tested using soil moisture probes-GS3.

**Key words:** porow media; infiltration; Suction head, Hydraulic Conductivity; abrupt profile

\* Part of Ph.D dissertation of the 1<sup>st</sup> author



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Received: 12/11/2023, Accepted: 11/2/2024, Published: 26/1/2026

### INTRODUCTION

**Mathematical analysis of the green and ampt equation:** Consider a vertical column of uniform porous medium of constant cross-sectional A and constant initial water content  $\theta_0$ . At time  $t=0$ , a constant depth of  $H$  of ponded water is applied instantaneously at the top end of the column,  $Z=0$ . Since  $\tau = \tau(\theta)$  implies that  $\theta = \theta(\tau)$  then  $K(\theta) = K[\theta(\tau)] = K(\tau)$ , and the Buckingham-Darcy flux relationship is :

$$q = K(\theta) [\partial\tau/\partial z] + K(\theta) \dots \quad (1)$$

Can be written as:

$$q = K(\tau) [\partial\tau/\partial z] + K(\tau) \dots \quad (2)$$

$$dt = (\theta_1 - \theta_0) L dL / [K_1 (H + \tau_w) + (K_1 - K_0) L] \dots \quad (14)$$

Integrating, and using the conditions  $L=0$  when  $t=0$ , yield.

Recalling that the two independent variables are time  $t$  and the positive downward position coordinate  $Z$ , we consider the infiltration process at fixed time, performing an

integration of Eq. (2) between limits  $Z=0$  and  $Z=Z_0$  to obtain:

$$\int_0^{Z_0} q dZ = \int_0^{Z_0} K(\tau) \left[ \frac{\partial \tau}{\partial Z} \right] dZ + \int_0^{Z_0} K(\tau) dZ \dots \dots \dots (3)$$

Where  $Z_0$  is depth at which  $\theta_1$  in its descent form  $\theta_1$  at  $Z=0$  along the general profile has just reached the initial value  $\theta_0$ . With  $t$  fixed for the integrations in Eq. (3), then  $(\partial \tau / \partial Z) dZ = d\tau$ . Also,  $\tau = -H$  at  $Z = 0$  and  $\tau = \tau(\theta_0) = \tau_0$  at  $Z = Z_0$ , so that Eq. (4) becomes

$$\int_0^{Z_0} q dZ = \int_{-H}^{\tau_0} K(\tau) d\tau + \int_0^{Z_0} K(\tau) dZ = \dots \dots \dots (4)$$

The first integral on the right-hand

side of Eq. (5) can be expressed as:

$$\int_{-H}^{\tau_0} K(\tau) d\tau = \int_{-H}^0 K(\tau) d\tau + \int_0^{\tau_0} K(\tau) d\tau \dots \dots \dots (5)$$

Over the suction head range  $-H \leq \tau \leq 0$ ,  $K(\tau)$  is fixed at the constant value  $K_1$ , so that the first integral on the right-hand side of Eq. (5) becomes simply  $K_1 H$ . Putting this result in Eq. (5) and combining with Eq. (4) yields:

$$\int_0^{Z_0} q dZ = K_1 H + \int_0^{\tau_0} K(\tau) d\tau + \int_0^{Z_0} K(\tau) dZ \dots \dots \dots (6)$$

Note that the validity of Eq. (6) is affected only by the constancy  $K(\tau) = K_1$ , over  $-H \leq \tau \leq 0$  and any question on that point is eliminated if  $K_1$  is the saturated hydraulic conductivity.

Note also that the first integral of the right-hand side of Eq. (6) is constant, because the integration limits are fixed (0 and  $\tau_0$ ) and  $K(\tau)$  is constant over  $0 \leq \tau \leq \tau_0$ .

Also, with  $\theta = \theta_1$  for  $0 \leq Z \leq L$ , then both  $q$  and  $K(\tau) = K_1$  become independent of  $Z$  in the first and last integrals, respectively, of equation (6), which then becomes:

$$q \int_0^L dZ = K_1 H + \int_0^{\tau_0} K(\tau) d\tau + K_1 \int_0^L dZ \dots \dots \dots (7)$$

Performing the simple integration of Eq. (7) and rearranging, yields

$$q = K_1 \left\{ H + K_1^{-1} \int_0^{\tau_0} K(\tau) d\tau + L \right\} / L \dots \dots \dots (8)$$

Returning now to the original Green and Ampt form, multiplying both sides of Eq. (1) by  $f$ . The term  $f dL/dt$  is the volumetric infiltration flux identifiable with Eq. (8).

The equivalence of Eq. (8) with Eq. (1) is completed by associating  $K_s$  with  $K_1$ , and  $\tau_w$  with the integral-containing term in Eq. (8), or

$$\tau_w = K_1^{-1} \int_0^{\tau_0} K(\tau) d\tau \dots \dots \dots (9)$$

The derivation leading to Eq. (9) has two very important aspects.

Firstly, a straightforward interpretation of  $\tau_w$  is provided by Eq. (9). It shows clearly that  $\tau_w$  is not simply equal to  $\tau_0$  and hence that (Youngs, 1974) contention of the physical unreality of  $\tau_w$  is unnecessary. In Particular, since  $K(\tau)$  decreases as  $\tau$  increases,  $\tau_w$  is always less than  $\tau_0$  generally much less for low  $\theta_0$  (High  $\tau_0$ ) and the most common shapes of  $K(\tau)$ . Equation (7) also shows how  $\tau_w$  will change with initial suction head  $\tau_0$ , and hence, because  $\tau_0 = \tau(\theta_0)$ , with initial water content  $\theta_0$ . For initially dry soils,  $\theta_0$  will be low,  $\tau_0$  will thus be high, and  $\tau_w$  will assume its maximum value... The reasonable circumstance for the maximum effect of capillarity.

To obtain relationships between  $L$ , cumulative time  $t$ , and the cumulative quantity  $I$  of infiltrated water, observe that the infiltration flux  $dy/dt = q$  of Eq. (8) is the sum of two components.

The first is  $(\theta_1 - \theta_0) dL/dt$ , the flow due to water being stored in the soil as the result of  $\theta$  changing in step-like fashion from  $\theta_0$  to  $\theta_1$  (Figure 1). The second is simply  $K_0$ , that is,  $K(\theta_0)$  times unit hydraulic gradient due to gravity. Therefore, as first written by (1):

$$q = (\theta_1 - \theta_0) dL / dt + K_0 = dI / dt \dots \dots \dots (10)$$

Separating the variable of Eq (11), integrating, and making use of the conditions  $I = 0$  and  $L = 0$  when  $t = 0$ , yields

$$I = (\theta_1 - \theta_0) L + K_0 t \dots \dots \dots (11)$$

Also, since  $di/dt = q$ , Eq. (8) and the first equality of Eq. (8) can be combined and rearranged so that

$$(\theta_1 - \theta_0) dL / dt = [K_1 (H + \tau_w) + (K_1 - K_0) L] / L \quad \dots \dots \dots \quad (12)$$

and the variables are separated to provide  
 $t = M^{-1}[u - F \ln(1 + u / F)] \quad \dots \dots \dots \quad (13)$

Where:

$$u = (\theta_1 - \theta_0) L \quad \dots \dots \dots \quad (14)$$

and the constants M, F, and are given by

$$M = K_1 - K_0 \quad \dots \dots \dots \quad (15)$$

$$F = K_1 a / (K_1 - K_0) \quad \dots \dots \dots \quad (16)$$

$$a = (\theta_1 - \theta_0) (H + \tau_w) \quad \dots \dots \dots \quad (17)$$

The term u in Eq (14) is the cumulative quantity of water stored in the soil as the water content  $\theta_0$  changes from  $\theta_0$  in step-like fashion (Fig. 1). Finally, Eq (10) and (15) can be combined to yield (Richards, 1954) (Aggelides and Youngs, 1978)

$$I - u = K_1 t \quad \dots \dots \dots \quad (18)$$

When the initial water content  $\theta_0$  is sufficiently small so that  $K_0 = K(\theta_0)$  is negligible, then  $M = K_1$ , from Eq (18) and  $u = I$  from Eq (20) so that Eq (14) becomes:

$$t = K_1^{-1}[I - a \ln(1 + I/a)] \quad \dots \dots \dots \quad (19)$$

Since  $u = I$ , Eq (17) can be used to write  $I = (\theta_1 - \theta_0) L$ , Which when used in Eq (19) yields the original integrated form of the Green and Ampt (1911) equation (Philip, 1958) (Philip, 1957a).

The objection of work reported here was to:

1-Rederive the Green and Ampt water infiltration Equation using an integral approach.

2-To test the Equation using laboratory soil uniform columns.

3-To compare the results of this Equation with the two-term Philip physically-based infiltration Equation.

## MATERIALS AND METHODS

**Studied traits:** To examine the suitability of the Green and Ampt equation with experimental data and compare it with

Binomial theorem of Philip equation, three different textured soils were selected: sandy, silt loam, and clay loam from various non-saline regions. The first soil sample was taken from the sediment of rivers in Babil Governorate (Al-Hindiya Dam), the second from the Jadriya site in Baghdad Governorate, and the third from a site in Islah area in Thi Qar Governorate. Soil samples were collected from horizon A (1-30 cm) after removing a 1 cm layer from the surface. These three soils were classified under the Typic Torrifluvents (Philip, 1957a). The soil samples taken from the field were air-dried under laboratory conditions and sieved with a 2 mm sieve opening, then stored under laboratory conditions. The soil samples were analyzed to determine their texture using the pipette method (Israelsen and Hansen, 1962). The bulk density of the soil at the three sites was measured using the metal cylinder core sampler method (Day, 1965) (Di, 2018). The Particle density was determined using the pycnometer method (Blake and Hartge) (Di, 2018), and the total porosity was calculated from the values of bulk density and particle density according to the method described by Vomocil (Day, 1965). Some chemical properties of the soil samples were measured, including electrical conductivity (E.C) of the 1:1 extract and using pH in the 1:1 extract using a pH meter, following the methods outlined by the U.S. Salinity Laboratory staff (Youngs, 1974). Cation exchange capacity and organic matter were determined using the wet digestion method (Welky and Black). Table (1) illustrates the results of the soil sample analyses used in the study.

**Table 1. Physical, Chemical and properties of the Soil**

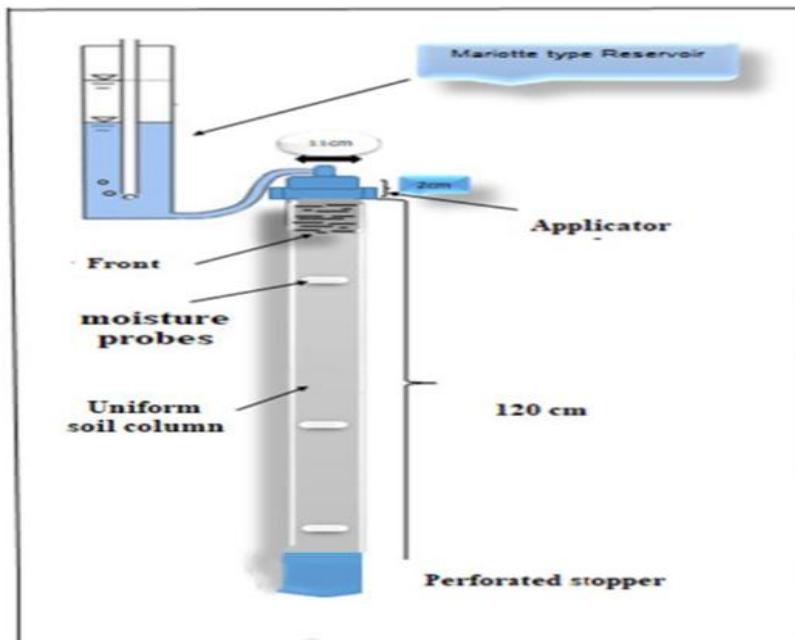
Soil Properties	Value	Value	Value	Unit
<b>bulk density</b>	1.30	1.4	1.6	Mg m <sup>-3</sup>
<b>Particle density</b>	2.61	2.65	2.65	Mg m <sup>-3</sup>
<b>porosity</b>	0.501	0.471	0.396	Cm <sup>3</sup> cm <sup>-3</sup>
<b>EC 1:1</b>	2.7	2.6	1.8	dS m <sup>-1</sup>
<b>pH 1:1</b>	7.1	7.3	7.3	-
<b>CEC</b>	23.8	9.89	3.1	Cmol kg <sup>-1</sup> soil
<b>Organic matter</b>	0.81	0.62	0.33	%
<b>Clay</b>	397	200	28.5	g kg <sup>-1</sup> soil
<b>Sand</b>	362	231	928.9	g kg <sup>-1</sup> soil
<b>Silt</b>	241	569	42.6	g kg <sup>-1</sup> soil
<b>Texture</b>	Clay loam	Silt loam	Sand	

In the study, cylindrical columns made of Plexiglas with a thickness of 0.5 cm, a length of 120 cm, and an inner diameter of 11 cm were used. The lower end of the column was sealed with a perforated piece of transparent organic glass with a thickness of 0.4 cm. These columns were filled with air-dried soil after determining their initial moisture content ( $\theta_0$ ). The soil was transferred into the soil column using another column of smaller diameter, the soil column. A funnel was used to place the soil into the central column, and the filling process proceeded as follows: the central column was lifted slightly by rotating it upwards, followed by a gentle pull of the soil column using the same rotational method. The lifting of the central columns was synchronized with tapping the soil column with a rubber hammer, tapping the outer wall of the column from bottom to top and from top to bottom while continuously rotating the soil column (Bouwer, 1967). It is ensured that the filling is uniform, whether for uniform columns or layers within the stratified columns, following the filling method described by (Bouwer, 1967), (Al-Douri, 2002). The homogeneity acceptance criterion for any column or layer within the column is that the coefficient of variation for the bulk density of the soil column does not exceed 2%. Three replicates for each soil are filled for uniform columns, resulting in 3 soils  $\times$  3 replicates = 9 uniform columns. The water applicator device was designed according to the specifications provided by (Al-Douri, 2002). It was manufactured from transparent

organic glass material, allowing easy visibility of water inside. It contains a rubber ring inside to prevent water leakage. It is connected to a valve for controlling the flow of water from the source. Several holes were designed at the center of the apparatus's end to facilitate connection with the top end of the columns. The purpose of this apparatus is to control and regulate the movement of water into the soil column in a closed system where the amount of water entering the soil is measured. Additionally, it prevents the dispersion and spreading of soil particles at the water entry surface, hindering control over the system during vertical water movement and under positive or negative hydraulic pressures (Masoud and Shahadh, 2022). The water infiltration experiment was conducted on the soil column, with a constant water pressure and a limit of 2 cm of water above the soil surface, using water supplied from a Mariotte-type reservoir. Water infiltration was measured in the soil columns after placing them vertically. The uniform columns were connected to the water applicator, and water was allowed to move through the soil body due to matric potential and gravitational force. Readings of the infiltrated water volume and the depth of the wetting front were taken over time. After the wetting front reached the end of the column, hydraulic conductivity was measured after the water continued flowing steadily from the lower end of the column. This was done by collecting a certain amount of water exiting from the bottom of the column over a specified time period and using Darcy's law. Soil moisture was measured before, during, and after the wetting front passed through three depths (20, 65, 100 cm) from the soil surface using GS3 moisture probes installed at those depths and stored using an Em50 data logger, as shown in Figure (1). After completing the infiltration measurements and water drainage from the column, The

soil was removed from the columns and was cut into sections to measure moisture content, bulk density, and Column uniform. This analysis was conducted using GS3 moisture

probes installed at various depths within the uniform columns during the vertical water flow for all experimental treatments, and the data were recorded using an Em50 data logger.



**Figure 1. Water infiltration experiment for the uniform columns**

## RESULTS AND DISCUSSION

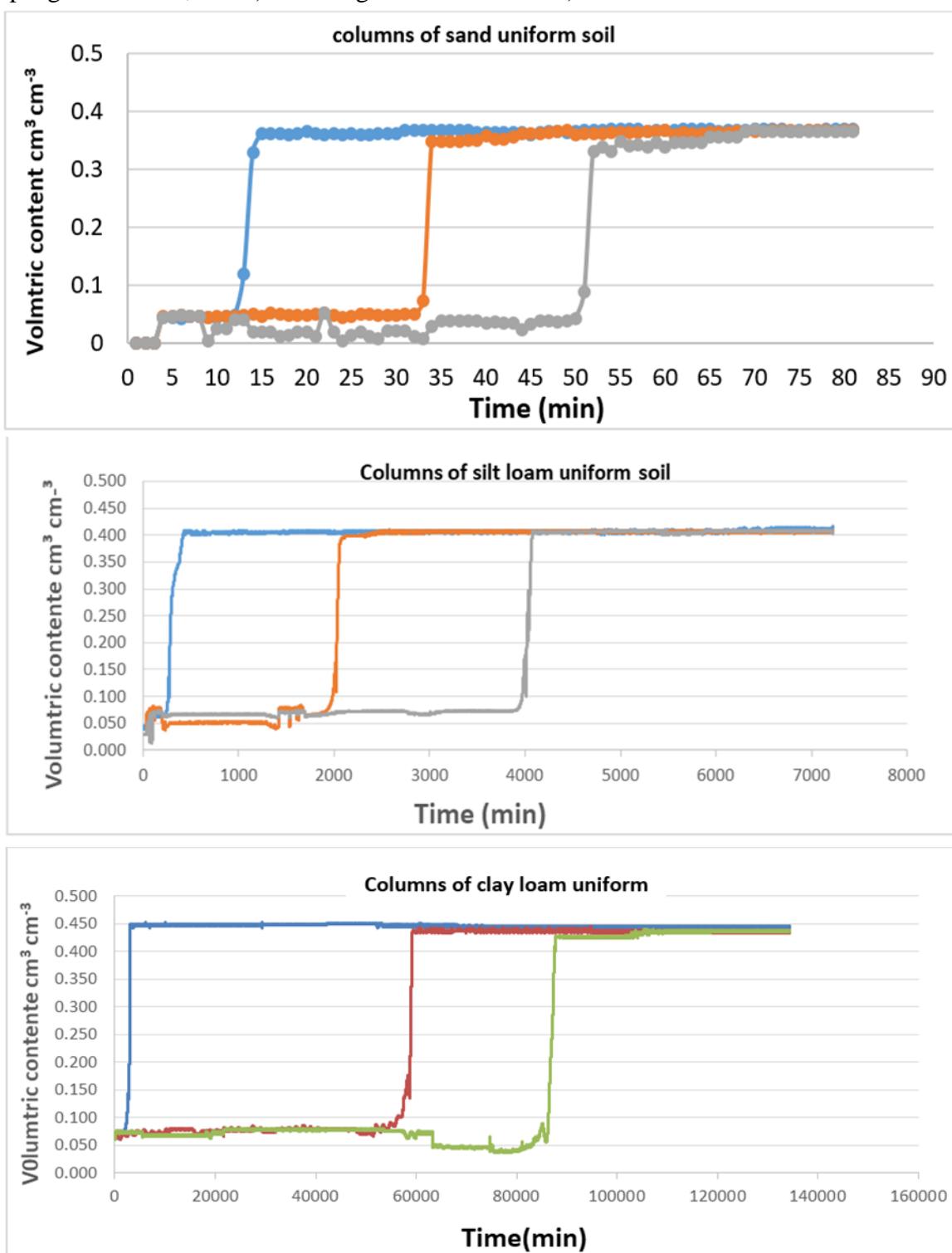
### Sharp front

Sharp wetting front sharpness was determined by analyzing the soil moisture content data before, during, and after the passage of the wetting front at a specific point in the column. The moisture profile curves resulting from the relationship between the advancement distance of the wetting front (cm) in vertically placed soil columns and the volumetric moisture content  $\theta$  ( $\text{cm}^3\text{cm}^{-3}$ ) varied with soil texture and the distance of the wetting front advance of 120 cm for the three soil types: sand, silt loam, and clay loam, at depths of moisture sensors (20, 65, and 100) cm, respectively. Figure (2) illustrates the profiles for uniform soil columns, which are consistent with the theoretical assumption of soil moisture profiles during infiltration. This aligns with the assumption of the Green and Ampt equation, which assumes piston. The moisture content becomes relatively uniform in distribution until reaching or near the wetting front, where

it sharply transitions between the wet and dry zones at the wetting front (Mohammed, and Salim ,2023) (Neuman1976) (Oudah and Aoda, 2022). The volumetric moisture content values extracted by the moisture sensors for uniform soil columns, sand, silt-loam, and clay-loam, had averages of 0.359, 0.401, and  $0.450 \text{ cm}^3\text{cm}^{-3}$ , respectively, at a depth of 120 cm. These averages were consistent with the volumetric moisture content values obtained the total porosity values for the same soil textures, which had averages of 0.396, 0.471, and  $0.501 \text{ cm}^3\text{cm}^{-3}$ , respectively, at a depth of 120 cm. The difference in volumetric moisture content and total porosity values varied significantly with soil texture. Uniform soil columns with fine texture achieved higher values of volumetric moisture content and total porosity, followed by those with medium texture, and then those with coarse texture. This is because increasing soil fineness leads to an increase in total porosity, enhancing the soil's ability to retain water in thin layers

covering soil particles and their aggregates. These results are consistent with what was mentioned by (Shokri and Salvucci, 2011) (Xiaopeng and Salifu, 2014). This aligns with

the assumption of the Green and Ampt equation, which assumes piston flow of the wetting front (Richards, 1954) (Al-Dulaimi, 1988).



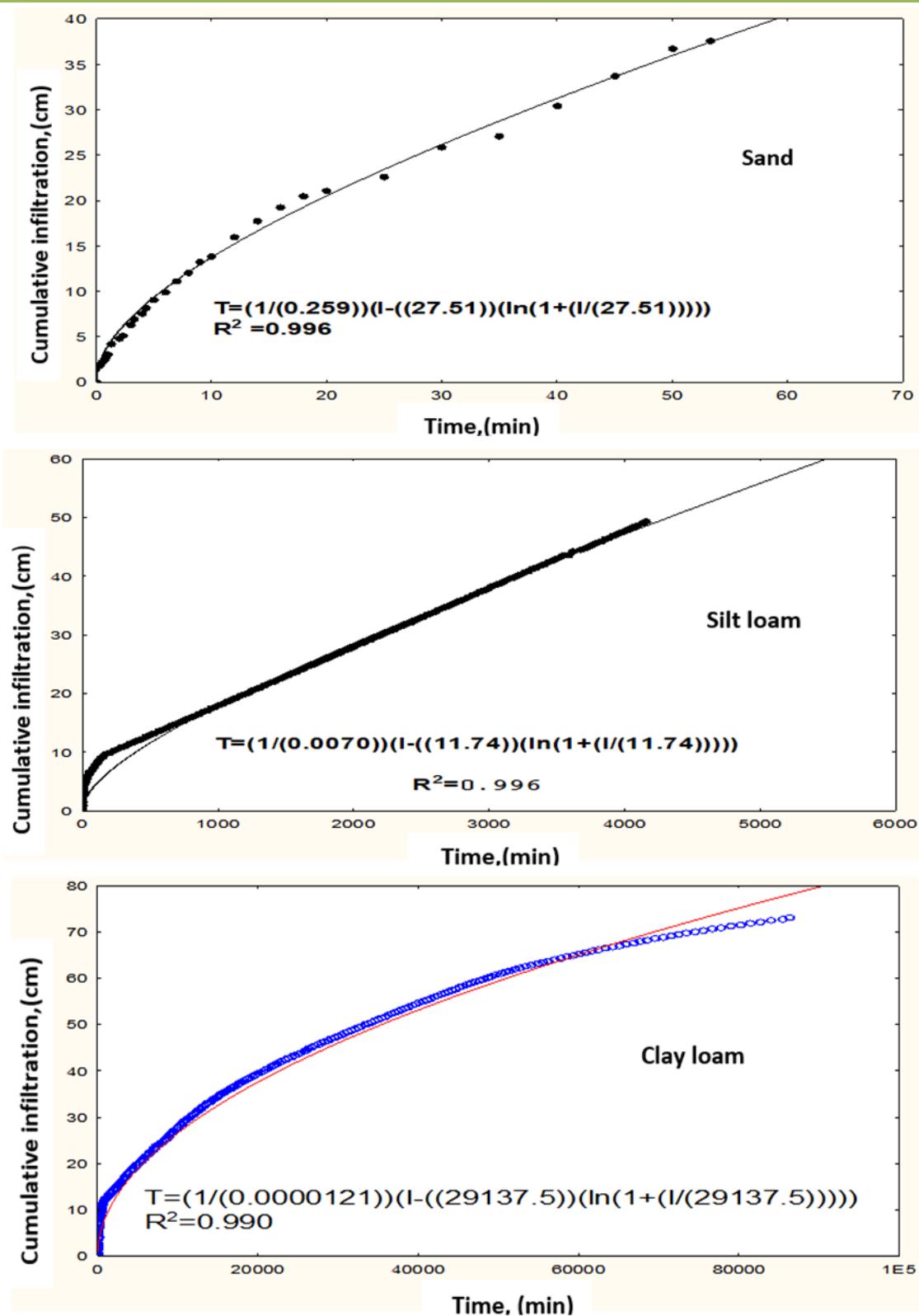
**Figure 2. The steepness of the wetland in uniform soil columns during vertical flow for a distance of 120 cm**

The moisture content becomes relatively uniform in distribution until reaching or near the wetting front, where it sharply transitions between the wet and dry zones at the wetting front (USDA, 2010) Xiaopeng and Salifu, 2014) (Younan, 2008).

### Vertical water infiltration

Green and Ampt equation was used to fit experimental data ( $I, t$ ) to describe infiltration for the three soil textures. Figure (3) after the wetting front reached a depth of 120 cm for the uniform columns, the infiltration data were matched with the Green and Ampt equation, where  $t$  represents the dependent variable and  $I$  represents the independent variable, meaning that  $t$  is a function of  $I$ . The time required for the wetting front to reach the end of the uniform soil column, at a distance of 120 cm, was 53.3, 3435, and 86339 minutes for the uniform sand, silt loam, and clay loam soils, respectively. The results indicate that the final time required to traverse a distance of 120 cm for uniform sand soil is less than that for silt loam and clay loam soils. This is attributed to the larger pore diameter in sandy soil (Masoud and Shahadh, 2022) (Mein, 1973) and the effect of gravitational forces on water movement in sand soil, as it has high hydraulic conductivity and low capillary properties, with large pores. Conversely, in soils with mixed textures like silt loam and clay loam, they exhibit high capillary properties and strong adhesive forces between soil particles and water, with a lower gravitational effect due to their low hydraulic conductivity. Additionally, smaller and medium-sized soil particles have a larger specific surface area than larger particles, leading to increased water film formation around soil particles and increased thickness. Porosity is one of the main properties determining and studying water infiltration time, where an increase in pore volume leads to an increase in the infiltration

rate, consistent with findings from various researchers (Kudayr and Salim, 2019) (Kawai, 2008). Figure (3) illustrates the relationship between cumulative infiltration ( $I$ ) and time ( $t$ ) during vertical water flow downwards. The values vary with different experimental treatments, as the total infiltrated water depth was 37.5, 49.3, and 73.0 cm for the uniform sand, silt loam, and clay loam soil treatments, respectively, at a depth of 120 cm. The cumulative vertical downward infiltration occurs due to the integration of the soil matric suction forces and (Mohammed and Salim, 2023) (Kacimov et al., 2010) (Shokri and Salvucci, 2011). It is evident from the above that the mathematical formula with a physical basis has been able to describe the depth of infiltrated water in uniform soil with different textures according to the Green and Ampt equation for vertical flow. Values of  $I$  were estimated using the matching criteria  $a$  and  $K_1$ . Table (2) shows the values of  $K_1$  and  $a$  extracted from the equation (19) matching with the experimental data  $I$  and  $t$ . By comparing the values of the constant  $K_1$  with the measured hydraulic conductivity, which was 0.28453, 0.00308, and 0.0000211  $\text{cm min}^{-1}$  for uniform sand, loam, and clayey soil columns, respectively. This convergence between the calculated and measured hydraulic conductivities confirms the validity of the match between the obtained values were close to the measured hydraulic conductivity (Shokri and Salvucci, 2011). The parameter ' $a$ ' can be used to estimate the suction value at the wetting front,  $\tau_w$ . As a result, the infiltration equation matched well with its tests for all uniform soil depths. The  $R^2$  values were very high for all uniform soils at a depth of 120 cm. The sandy uniform soil yielded an  $R^2$  value of 0.995, the loamy mix had an  $R^2$  value of 0.999, while the clay loam reached an  $R^2$  value of 0.996.



**Figure 3. Cumulative infiltration in uniform soil columns during vertical water flow over a distance of 120 cm using the Green and Ampt equation**

All of these values are statistically significant at a confidence level of 1%. The ranking of uniform soils in terms of matching preference

is as follows: silty loamy, clayey loam, then sandy. This is also illustrated in Table 2.

**Table 2. Fittings Green and Ampt equation with experimental data for three different textured soil columns (120 cm).**

Texture	$K_1$ Cm min <sup>-1</sup>	a cm	Green and Ampt		
			Coefficient of Determination $R^2$	$K_1$ Measures cm min	$\tau_w$ Cm
Sand	0.2592	27.516	0.995**	0.28453	53.3
Silt loam	0.0029	69.87	0.999**	0.00308	3435
Clay loam	0.00001210	29137.5	0.996**	0.0000211	86339

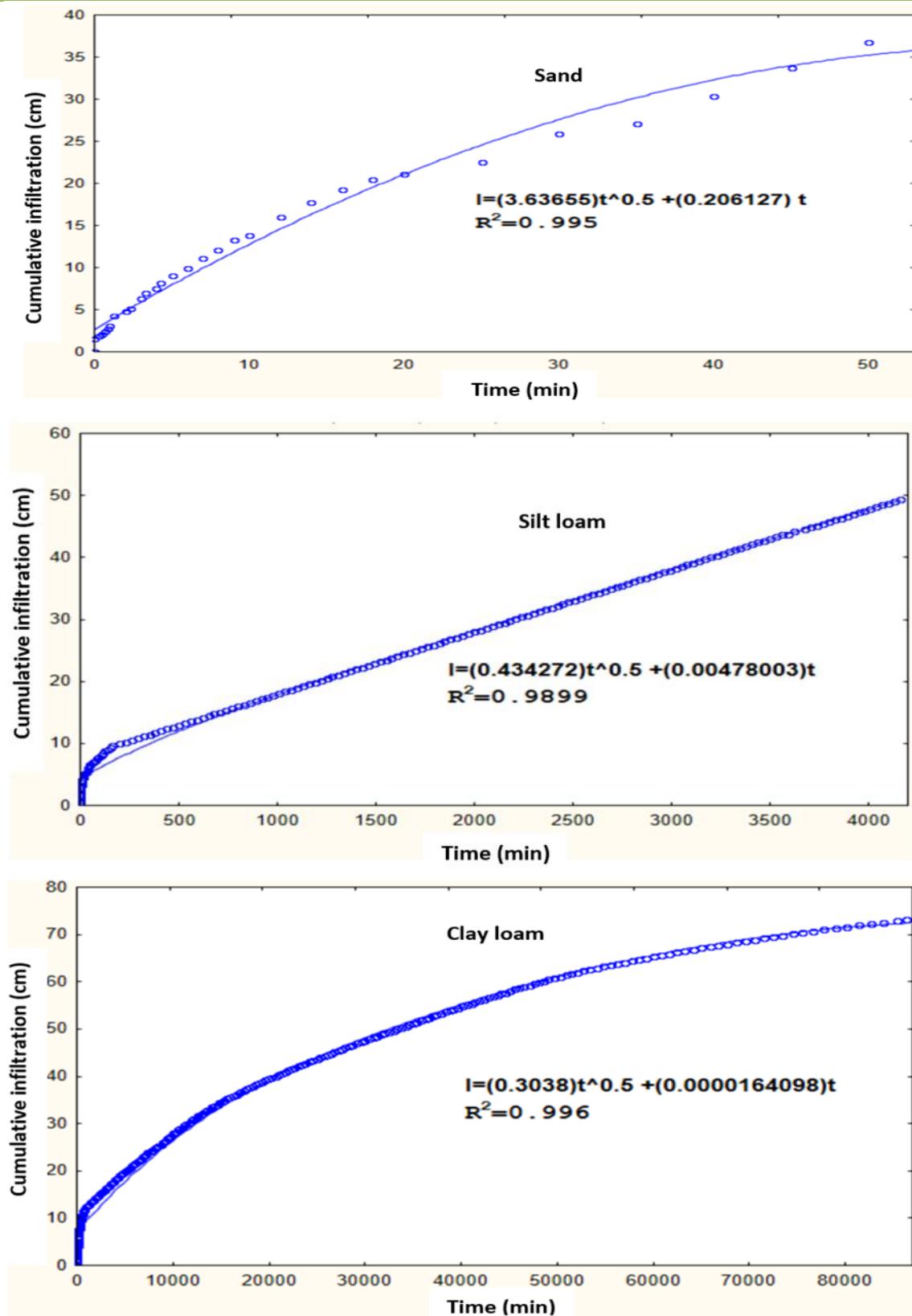
**Comparison between the Green and Ampt equation (1911) and the Philip equation (1957a) for describing infiltration data**

As mentioned earlier, the Green and Ampt equation has shown very good matching with experimental data. Now, it remains to compare this equation with another equation based on physical principles, namely the Philip equation (1957a), which is a two-term equation. From Table (3), and based on the values of  $R^2$  (the coefficient of determination), it is evident that the Green and Ampt equation provides a better match than the Philip equation for silty loamy soil columns. (Shwetha, et al., 2013). However, both the Philip and Green and Ampt equations showed equal matching with clay

loam and sand soils, with  $R^2$  values. When comparing the values of  $K_1$  in the Green and Ampt equation and the values of A in the Philip equation, which represent near-saturated hydraulic conductivity values, (Bazargani, 1964) (Kawai, 2008) (Zhao, et al., 2018) it is observed that the  $K_1$  values in the Green and Ampt equation are higher than the A values in the Philip equation for uniform loam soils at a depth of 120 cm. While, the measured hydraulic conductivity was close to the estimated hydraulic conductivity values of the Green and Ampt equation, with an average of 0.2845, 0.00308, 0.000021 cm min<sup>-1</sup> for sandy, silty loam, and clay loam soil columns respectively, as shown in Figure (4).

**Table 3. Fittings Philip two term equation with infiltration data of three different soil texture columns**

Texture	equation Philip		
	S Cm min <sup>-1/2</sup>	A cm min <sup>-1</sup>	Coefficient of Determination $R^2$
Sand	3.636	0.206	0.995**
Silt loam	0.434	0.004	0.989**
Clay loam	0.303	0.000016	0.996**



**Figure 4.** Cumulative infiltration in uniform soil columns during vertical water flow using the Philip equation

## CONCLUSION

The performance of the **Green and Ampt (1911)** equation was excellent (with an error rate of less than 1%) in describing water infiltration data in both homogeneous and layered soils.

The **Green and Ampt (1911)** equation was re-derived to make it suitable for describing infiltration data in layered soils with any desired number of layers.

The **hydraulic conductivity** and the **suction at the wetting front** were then estimated using the Green and Ampt equation, based on criteria involving the variation rate according to the soil texture and its position within the layered system.

Using **GS3 moisture sensors** to measure soil moisture at different depths in the field showed that the movement of soil water during infiltration follows a **piston-type flow system**, which is fundamentally one of the assumptions of the Green and Ampt equation.

## ACKNOWLEDGEMENT

I would like to express my sincere gratitude to everyone who contributed to the completion of this research and to its presentation in its current form.

My deepest appreciation goes to Dr. Mahdi Oudah Al-Tamimi for his valuable guidance and constructive feedback, which had a significant impact on the development of this work. I also extend my thanks to the faculty of the College of Agricultural Engineering Sciences, University of Baghdad, for providing a supportive and stimulating academic environment.

## CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

## DECLARATION OF FUND

The authors declare that they have not received a fund.

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Author/s signature on Ethical Approval Statement.

Ethical Clearance and Animal welfare

Funds:

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## طريقة التكامل لأعادة اشتقاق معادلة Green and Ampt لغيب الماء وتقويمها تجربياً

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### المستخلص

قيمت طريقة (1911) Green and Ampt لوصف نفوذ الماء في الوسط المسامي بدرجة حرجة ، اذ تمت مراجعة النظرية للأشتقاق الرياضي السابق وذلك لأعطاءها سند قوي لوصف الماء الجاري في مقد التربة ذي الشكل المستطيل الحاد . تم اشتقاق هذه المعادلة وانت تعبير مباشر لعمود السحب عند جبهة الأبتلال  $w$ . ان عمود السحب عند جبهة الأبتلال  $w$  وضع كتكامل لعمود السحب  $a$  المعتمد على تكامل دالة الأيقالية المائية المعتمدة على عمود السحب  $a$  مع كون النهاية العليا للتكامل هي عمود السحب  $a$  للمحتوى الرطوبى الابتدائي  $a_0$  لذلك فأن شكل  $w$  المعتمد على  $a_0$  موضح في هذا التكامل . يضاف الى ذلك فأن  $w$  يكون ثابت لعملية الجريان حالة الشكل المستطيلي الحاد يتضح انه الأيقالية المائية  $(\theta_1)$   $K$  للمنطقة الرطبة خلف جبهة الأبتلال والتي هي دالة لمتوسط المحتوى الرطوبى الثابت للمنطقة الرطبة خلف جبهة الأبتلال. بالنسبة للبيانات التجريبية للغيب التراكمي مع الزمن تمت مطابقتها  $G-A$  و  $Philip$  الغير الخطية. اعطت مطابقة معادلة  $G-A$  بشكل افضل من مطابقة معادلة  $Philip$  لتراب معينة ، ولكن كلا المعادلين اعطت مطابقة جيدة جداً (خطا 1%). اعطى المعيار  $K_1$  لمعادلة  $G-A$  تطابق جيد جداً مع الأيقالية المائية المقاسة لجميع الترب التي تمت دراستها . تم حساب  $w$  من معيار  $a$  لتراب مختلفة النسجة . قيس المحتوى الرطوبى الحجمي لمقد التربة بواسطة  $GS_3$ .

الكلمات المفتاحية: الوسط المسامي، الغيب، الأيقالية المائية المشبعة ، انحدار مقد تربة.